Lesson 6-1

Example 1

a. Change 240° to radian measure in terms of π.

$240° = 240° \times \frac{\pi}{180°}$ *1 degree =* $\frac{\pi}{180°}$

$= \frac{4\pi}{3}$

b. Change $\frac{3\pi}{4}$ radians to degree measure.

$\frac{3\pi}{4} = \frac{3\pi}{4} (\frac{180°}{\pi}$ *1 radian =* $\frac{180°}{\pi}$

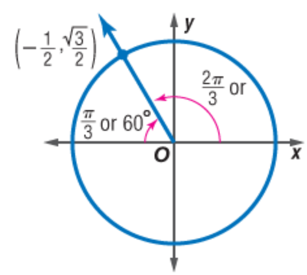
$= 135°$

Example 2

Evaluate cos $\frac{2\pi}{3}$.

The reference angle for $\frac{2\pi}{3}$ is $\frac{\pi}{3}$.

Since $\frac{\pi}{3} = 60°$, the terminal side of the angle intersects the unit circle at a point with coordinates of $1\frac{1}{2}, \frac{\sqrt{3}}{2}2$. Because the terminal side of the angle is in the second quadrant, the *x*-coordinate is negative. The point of intersection has coordinates $1-\frac{1}{2}, \frac{\sqrt{3}}{2}2$.

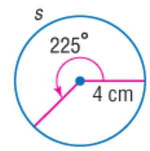


Therefore, cos $\frac{2\pi}{3} = -\frac{1}{2}$.

Example 3

Given a central angle of 225°, find the length of its intercepted arc in a circle of radius 4 centimeters. Round to the nearest tenth.

First, convert the measure of the central angle from degrees to radians.



$225° = 225° \left(\frac{\pi}{180°}\right.$ *1 degree* $= \frac{\pi}{180°}$

$= \frac{5}{4}\pi$ or $\frac{5\pi}{4}$

Then, find the length of the arc.

$s = r\theta$

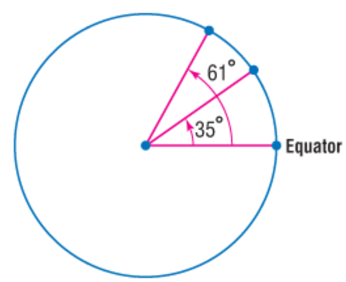
$s = 41\frac{5\pi}{4}2$ $r = 4,\ \theta = \frac{5\pi}{4}$

$s = 15.70796327$ *Use a calculator.*

The length of the arc is about 15.7 centimeters.

Example 4

GEOGRAPHY **Two cities lie along the same longitude line. The latitude of the first city is 61° N and the latitude of the second city is 35° N. The radius of Earth is about 3960 miles. Find the approximate distance between the two cities.**



The length of the arc between the two cities is the distance between the two cities. The measure of the central angle subtended by this arc is 61° - 35° or 26°.

$26° = 26° \times \frac{\pi}{180°}$ *1 degree* $= \frac{\pi}{180°}$

$= \frac{13\pi}{90}$

$s = r\theta$

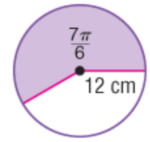
$s = 39601\frac{13\pi}{90}2$ $r = 3960,\ \theta = \frac{13\pi}{90}$

$s \approx 1796.990998$

The distance between the two cities is about 1797 miles.

Example 5

Find the area of a sector if the central angle measures $\frac{7\pi}{6}$ radians and the radius of the circle is 12 centimeters. Round to the nearest tenth.



$A = \frac{1}{2}r^2\theta$ *Formula for the area of a circular sector*

$A = \frac{1}{2}(12^2)\left(\frac{7\pi}{6}\right)$ *$r = 12$, $\theta = \frac{7\pi}{6}$*

$A \approx 263.8937829$

The area of the sector is about 263.9 square centimeters.

Example 6

BUSINESS **The Morgan Sporting Goods Company manufactures colored parachutes to be used in physical education classes at elementary schools. An order is received for fifteen 8-foot diameter parachutes. Each of the parachutes is divided into 8 sections, each made from a different color of fabric. How much fabric of each color will be needed to fill the order?**

There are 2π radians in a complete circle and 8 equal sections or sectors in the parachute. Therefore, the measure of each central angle is $\frac{2\pi}{8}$ or $\frac{\pi}{4}$ radians. If the diameter of the circle is 8 feet, the radius is 4 feet. Use these values to find the area of each sector.

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(4^2)\left(\frac{\pi}{4}\right)$

$A \approx 6.283185307$ *Use a calculator.*

Since there are fifteen parachutes, multiply the area of each sector by 15. The Morgan Sporting Goods Company needs about 94.2 square feet of each color of fabric.
