

Study Guide

Linear and Angular Velocity

As a circular object rotates about its center, an object at the edge moves through an angle relative to the object's starting position. That is known as the **angular displacement**, or angle of rotation. **Angular velocity** ω is given by $\omega = \frac{\theta}{t}$, where θ is the angular displacement in radians and t is time. **Linear velocity** v is given by $v = r\frac{\theta}{t}$, where $\frac{\theta}{t}$ represents the angular velocity in radians per unit of time. Since $\omega = \frac{\theta}{t}$, this formula can also be written as $v = r\omega$.

Example 1 Determine the angular displacement in radians of 3.5 revolutions. Round to the nearest tenth.

Each revolution equals 2π radians. For 3.5 revolutions, the number of radians is $3.5 \times 2\pi$, or 7π . 7π radians equals about 22.0 radians.

Example 2 Determine the angular velocity if 8.2 revolutions are completed in 3 seconds. Round to the nearest tenth.

The angular displacement is $8.2 \times 2\pi$, or 16.4π radians.

$$\omega = \frac{\theta}{t}$$

$$\omega = \frac{16.4\pi}{3} \qquad \theta = 16.4\pi, t = 3$$

$$\omega \approx 17.17403984 \qquad \text{Use a calculator.}$$

The angular velocity is about 17.2 radians per second.

Example 3 Determine the linear velocity of a point rotating at an angular velocity of 13π radians per second at a distance of 7 centimeters from the center of the rotating object. Round to the nearest tenth.

$$v = r\omega$$

$$v = 7(13\pi) \qquad r = 7, \omega = 13\pi$$

$$v \approx 285.8849315 \qquad \text{Use a calculator.}$$

The linear velocity is about 285.9 centimeters per second.

Practice

Linear and Angular Velocity

Determine each angular displacement in radians. Round to the nearest tenth.

1. 6 revolutions
2. 4.3 revolutions
3. 85 revolutions
4. 11.5 revolutions
5. 7.7 revolutions
6. 17.8 revolutions

Determine each angular velocity. Round to the nearest tenth.

7. 2.6 revolutions in 6 seconds
8. 7.9 revolutions in 11 seconds
9. 118.3 revolutions in 19 minutes
10. 5.5 revolutions in 4 minutes
11. 22.4 revolutions in 15 seconds
12. 14 revolutions in 2 minutes

Determine the linear velocity of a point rotating at the given angular velocity at a distance r from the center of the rotating object. Round to the nearest tenth.

13. $\omega = 14.3$ radians per second, $r = 7$ centimeters
14. $\omega = 28$ radians per second, $r = 2$ feet
15. $\omega = 5.4\pi$ radians per minute, $r = 1.3$ meters
16. $\omega = 41.7\pi$ radians per second, $r = 18$ inches
17. $\omega = 234$ radians per minute, $r = 31$ inches
18. **Clocks** Suppose the second hand on a clock is 3 inches long. Find the linear velocity of the tip of the second hand.