Lesson 6-4

Example 1

a. **State the amplitude for the function $y = 3 \cos \theta$.**

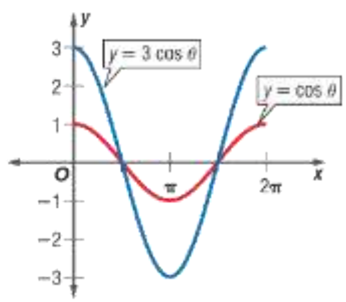
b. **Graph $y = 3 \cos \theta$ and $y = \cos \theta$ on the same set of axes.**

c. **Compare the graphs.**

a. According to the definition of amplitude, the amplitude of $y = A \cos \theta$ is $|A|$. So, the amplitude of $y = 3 \cos \theta$ is $|3|$ or 3.

b. Make a table of values. Then graph the points and draw a smooth curve.

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
cos θ	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1
3 cos θ	3	$\frac{3\sqrt{2}}{2}$	0	$-\frac{3\sqrt{2}}{2}$	-3	$-\frac{3\sqrt{2}}{2}$	0	$\frac{3\sqrt{2}}{2}$	3



c. Both graphs cross the axis at the same points and also reach the minimum and maximum values at the same points. The difference is that the minimum and maximum values of $y = \cos \theta$ are -1 and 1, and the minimum and maximum values of $y = 3 \cos \theta$ are –3 and 3.

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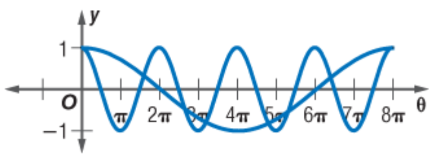
Example 2

a. **State the period for the function $y = \cos \frac{\theta}{4}$.**

b. **Graph $y = \cos \frac{\theta}{4}$ and $y = \cos \theta$.**

a. The definition of the period of $y = \cos k\theta$ is $\frac{2\pi}{k}$. So, the period for the function $y = \cos \frac{\theta}{4}$ is $\frac{2\pi}{\frac{1}{4}}$ or 8π.

b.

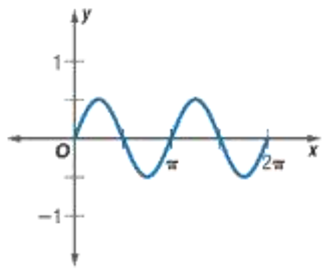


Example 3

State the amplitude and period for the function $y = \frac{1}{2} \sin 2\theta$. Then graph the function.

Since $A = \frac{1}{2}$, the amplitude is the absolute value of A or $\frac{1}{2}$. Since $k = 2$, the period is $\frac{2\pi}{2}$ or π.

Use the basic shape of the sine function and the amplitude and period to graph the equation.



Example 4

Write an equation of the cosine function with amplitude 4.5 and period 8π.

The form of the equation will be $y = A \cos k\theta$. First, find the possible values of A for an amplitude of 4.5.

$$|A| = 4.5$$
$$A = 4.5 \text{ or } -4.5$$

Since there are two values of A, two possible equations exist.

Now find the value of k when the period is 8π.

$$\frac{2\pi}{k} = 8\pi$$ *The period of a cosine function is $\frac{2\pi}{k}$.*

$$k = \frac{2\pi}{8\pi} \text{ or } \frac{1}{4}$$

The possible equations are $y = 4.5 \cos \frac{1}{4}\theta$ or $y = -4.5 \cos \frac{1}{4}\theta$.

Example 5

PHYSICS The motion of a weight on a certain kind of spring can be described by a modified trigonometric function. At time 0, Carrie pushes the weight upward 3 inches from its equilibrium point and releases it. She finds that the weight returns to the point three inches above the equilibrium point after 2 seconds.

a. **Write an equation for the motion of the weight.**

b. **What will the position of the weight be after 15.5 seconds?**

a. At time 0, the weight is 3 inches above the equilibrium point and at its maximum value. Since the weight will fall, the values will get smaller. The function will be a cosine function with a positive value of A.

The maximum and minimum points are 3 and –3. Thus, the amplitude, A, is 3.

The weight makes a complete cycle is 2 seconds. Thus, the period is 2.

$$\frac{2\pi}{k} = 2$$
$$k = \pi$$

Now, write the equation.

$y = A \cos kt$
$y = 3 \cos \pi t$

b. Use a calculator to find the value of y when $t = 15.5$ to find the position of the weight after 15.5 seconds.

$y = 3 \cos (15.5\pi)$
$y = 0$

After 15.5 seconds, the weight will be at the equilibrium point.

Example 6
MUSIC Write an equation of the sine function that represents the initial behavior of the vibrations of the note G above middle C having amplitude 0.009 and a frequency of 420 hertz.

The general form of the equation will be $y = A \sin kt$, where t is the time in seconds. Since the amplitude is 0.009, $A = \pm\, 0.009$.

The period is the reciprocal of the frequency or $\frac{1}{420}$. Use this value to find k.

$$\frac{2\pi}{k} = \frac{1}{420}$$

$$k = 2\pi(420) \text{ or } 840\pi$$

One sine function that represents the vibration is $y = 0.009\sin(840\pi \times t)$.