

## Lesson 6-5

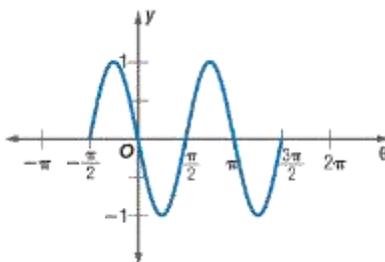
**Example 1**

State the phase shift for each function. Then graph the function.

a.  $y = \sin(2\theta + \pi)$

The phase shift of the function is  $-\frac{c}{k}$  or  $-\frac{\pi}{2}$ .

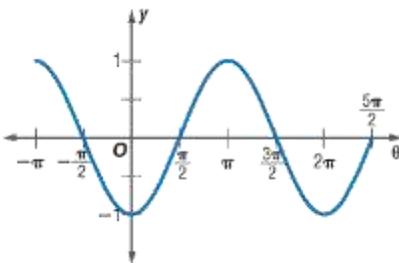
To graph  $y = \sin(2\theta + \pi)$ , consider the graph of  $y = \sin 2\theta$ . Graph this function and then shift the graph  $-\frac{\pi}{2}$ .



b.  $y = \cos(\theta - \pi)$

The phase shift of the function is  $-\frac{c}{k}$  or  $-\frac{-\pi}{1}$ , which equals  $\pi$ .

To graph  $y = \cos(\theta - \pi)$ , consider the graph of  $y = \cos \theta$  and then shift the graph  $\pi$ .



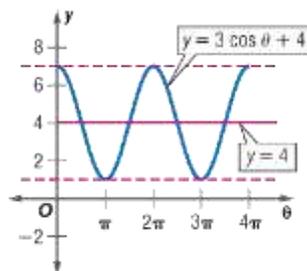
**Example 2**

State the vertical shift and the equation of the midline for the function  $y = 3 \cos \theta + 4$ .

Then graph the function.

The vertical shift is 4 units upward. The midline is the graph  $y = 4$ .

To graph the function, draw the midline, the graph of  $y = 4$ . Since the amplitude of the function is 3, draw dashed lines parallel to the midline which are 3 units above and below the midline. Then draw the cosine curve.

**Example 3**

State the amplitude, period, phase shift, and vertical shift for  $y = 2 \cos \frac{\theta}{2} + \pi + 3$ .

The amplitude is  $|2|$  or 2.

The period is  $\frac{2\pi}{\frac{1}{2}}$  or  $4\pi$ . The phase shift is  $-\frac{\pi}{\frac{1}{2}}$  or  $-2\pi$ .

The vertical shift is  $+3$ .

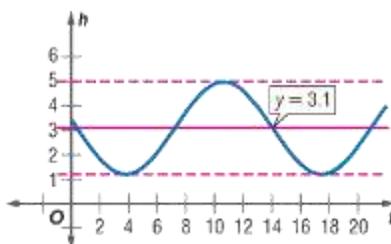
**Example 4**

**TIDES** The equation that models the tides off the coast of a city on the east coast of the United States is given by  $h = 3.1 + 1.9 \sin 1\frac{\pi}{6.8}t - \frac{5.1\pi}{6.8}$ , where  $t$  represents the number of hours since midnight and  $h$  represents the height of the water. Draw a graph that models the cyclic nature of the tide.

The vertical shift is 3.1. Draw the midline  $y = 3.1$ . The amplitude is 1.9. Draw dashed lines parallel to and 1.9 units above and below the midline.

The period is  $\frac{2\pi}{\frac{\pi}{6.8}}$  or 13.6. Draw the sine curve with a period of 13.6.

Shift the graph  $-\frac{5.1\pi}{6.8}$  or 5.1 units.



**Example 5**

Write an equation of a sine function with amplitude 5, period  $3\pi$ , phase shift  $\frac{\pi}{2}$ , and vertical shift 2.

The form of the equation will be  $y = A \sin(k\theta + c) + h$ . Find the values of  $A$ ,  $k$ ,  $c$ , and  $h$ .

$$\begin{aligned} \mathbf{A:} \quad |A| &= 5 \\ A &= 5 \text{ or } -5 \end{aligned}$$

$$\begin{aligned} \mathbf{k:} \quad \frac{2\pi}{k} &= 3\pi && \textit{The period is } 3\pi \\ k &= \frac{2}{3} \end{aligned}$$

$$\begin{aligned} \mathbf{c:} \quad -\frac{c}{k} &= \frac{\pi}{2} && \textit{The phase shift is } \frac{\pi}{2}. \\ -\frac{c}{\frac{2}{3}} &= \frac{\pi}{2} && k = \frac{2}{3} \\ c &= -\frac{\pi}{3} \end{aligned}$$

$$\mathbf{h:} \quad h = 2$$

Substitute these values into the general equation. The possible equations are

$$y = 5 \sin \left( \frac{2}{3}\theta - \frac{\pi}{3} \right) + 2 \text{ or } y = -5 \sin \left( \frac{2}{3}\theta - \frac{\pi}{3} \right) + 2 .$$

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**Example 6****Graph  $y = x + \sin x$ .**

First graph  $y = x$  and  $y = \sin x$  on the same axes. Then add the corresponding ordinates of the functions. Finally, sketch the graph.

$x$	$\sin x$	$x + \sin x$
0	0	0
$\frac{\pi}{2}$	1	$\frac{\pi}{2} + 1 \approx 2.57$
$\pi$	0	$\pi \approx 3.14$
$\frac{3\pi}{2}$	-1	$\frac{3\pi}{2} - 1 \approx 3.71$
$2\pi$	0	$2\pi \approx 6.28$
$\frac{5\pi}{2}$	1	$\frac{5\pi}{2} + 1 \approx 8.85$
$3\pi$	0	$3\pi \approx 9.42$

