

Study Guide

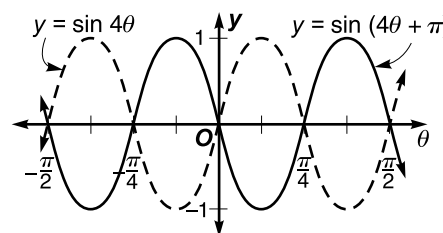
Translations of Sine and Cosine Functions

A horizontal translation of a trigonometric function is called a **phase shift**. The phase shift of the functions $y = A \sin(k\theta + c)$ and $y = A \cos(k\theta + c)$ is $-\frac{c}{k}$, where $k > 0$. If $c > 0$, the shift is to the left. If $c < 0$, the shift is to the right. The **vertical shift** of the functions $y = A \sin(k\theta + c) + h$ and $y = A \cos(k\theta + c) + h$ is h . If $h > 0$, the shift is upward. If $h < 0$, the shift is downward. The **midline** about which the graph oscillates is $y = h$.

Example 1 State the phase shift for $y = \sin(4\theta + \pi)$. Then graph the function.

The phase shift of the function is $-\frac{c}{k}$ or $-\frac{\pi}{4}$.

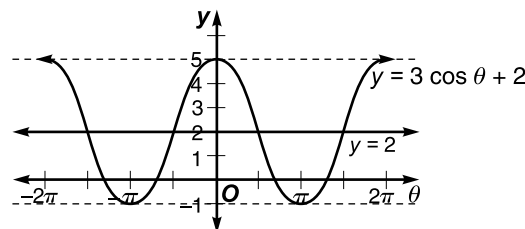
To graph $y = \sin(4\theta + \pi)$, consider the graph of $y = \sin 4\theta$. The graph of $y = \sin 4\theta$ has an amplitude of 1 and a period of $\frac{\pi}{2}$. Graph this function, then shift the graph $-\frac{\pi}{4}$.



Example 2 State the vertical shift and the equation of the midline for $y = 3 \cos \theta + 2$. Then graph the function.

The vertical shift is 2 units upward. The midline is the graph of $y = 2$.

To graph the function, draw the midline. Since the amplitude of the function is $|3|$, or 3, draw dashed lines parallel to the midline which are 3 units above and below $y = 2$. That is, $y = 5$ and $y = -1$. Then draw the cosine curve with a period of 2π .



Example 3 Write an equation of the cosine function with amplitude 2.9, period $\frac{2\pi}{5}$, phase shift $-\frac{\pi}{2}$, and vertical shift -3 .

The form of the equation will be $y = A \cos(k\theta + c) + h$. Find the values of A , k , c , and h .

$$\mathbf{A:} \quad |A| = 2.9$$

$$A = 2.9 \text{ or } -2.9$$

$$\mathbf{k:} \quad \frac{2\pi}{k} = \frac{2\pi}{5} \quad \text{The period is } \frac{2\pi}{5}.$$

$$k = 5$$

$$\mathbf{c:} \quad -\frac{c}{k} = -\frac{\pi}{2}$$

$$-\frac{c}{5} = -\frac{\pi}{2}$$

$$c = \frac{5\pi}{2}$$

$$\mathbf{h:} \quad h = -3$$

$$\text{The phase shift is } -\frac{\pi}{2}.$$

$$k = 5$$

The possible equations are $y = \pm 2.9 \cos\left(5\theta + \frac{5\pi}{2}\right) - 3$.

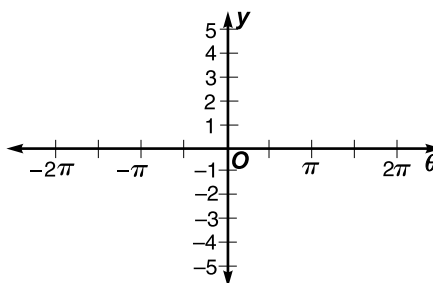
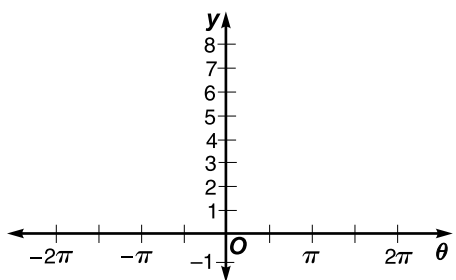
Practice

Translations of Sine and Cosine Functions

State the vertical shift and the equation of the midline for each function. Then graph each function.

1. $y = 4 \cos \theta + 4$

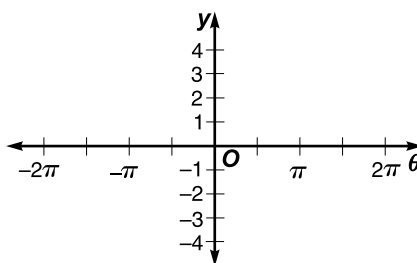
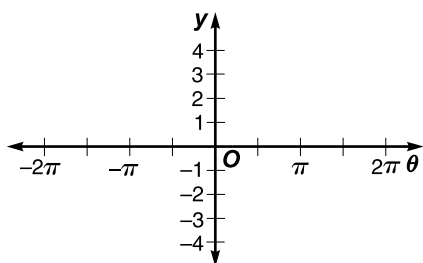
2. $y = \sin 2\theta - 2$



State the amplitude, period, phase shift, and vertical shift for each function. Then graph the function.

3. $y = 2 \sin \left(\theta + \frac{\pi}{2} \right) - 3$

4. $y = \frac{1}{2} \cos (2\theta - \pi) + 2$



Write an equation of the specified function with each amplitude, period, phase shift, and vertical shift.

5. sine function: amplitude = 15, period = 4π , phase shift = $\frac{\pi}{2}$, vertical shift = -10

6. cosine function: amplitude = $\frac{2}{3}$, period = $\frac{\pi}{3}$, phase shift = $-\frac{\pi}{3}$, vertical shift = 5

7. sine function: amplitude = 6, period = π , phase shift = 0, vertical shift = $-\frac{3}{2}$