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Example 1

METEOROLOGY The table contains the times that the sun rises and sets on the fifteenth of every month in Center City.

Month	Sunrise A.M.	Sunset P.M.
January	7:15	5:55
February	7:00	6:27
March	6:45	6:43
April	6:10	6:55
May	5:45	7:10
June	5:35	7:28
July	5:50	7:30
August	6:05	7:10
September	6:20	6:40
October	6:30	6:07
November	6:50	5:45
December	7:10	5:42

Let $t = 1$ represent January 15.
Let $t = 2$ represent February 15.
Let $t = 3$ represent March 15.

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a. Write a function that models the hours of daylight for Center City.

b. Use your model to estimate the number of hours of daylight on May 30.

a. First compute the amount of daylight for each day as a decimal value. Consider January 15.

First, write each time in 24-hour time.

7:15 A.M. = 7:15
5:55 P.M. = 5:55 + 12 or 17:55

Then change each time to a decimal rounded to the nearest hundredth.

$$7{:}15 = 7 + \frac{15}{60} \text{ or } 7.25$$

$$17{:}55 = 17 + \frac{55}{60} \text{ or } 17.92$$

On January 15, there will be 17.92 – 7.25 or 10.67 hours of daylight.

Similarly, the number of daylight hours can be determined for the fifteenth of each month.

Month	Jan	Feb	Mar	Apr	May	June
t	1	2	3	4	5	6
Hours of Daylight	10.67	11.45	11.97	12.75	13.42	13.89

Month	July	Aug	Sept	Oct	Nov	Dec
t	7	8	9	10	11	12
Hours of Daylight	13.67	13.09	12.34	11.62	10.92	10.53

The data can be modeled by a function of the form $y = A \sin(kt + c) + h$, where t is the time in months. First, find A, h, and k.

A: $A = \frac{13.89 - 10.53}{2}$ *A is half the difference between the most*

$= 1.68$ *daylight (13.89 h) and the least daylight (10.53 h).*

h: $h = \frac{13.89 + 10.53}{2}$ *h is half the sum of the greatest value and*

$= 12.21$ *the least value.*

k: $\frac{2\pi}{k} = 12$ *The period is 12.*

$k = \frac{\pi}{6}$

Substitute these values into the general form of the sinusoidal function.

$y = A \sin(kt + c) + h$

$y = 1.68 \sin\left(\frac{\pi}{6}t + c\right) + 12.21$ *$A = 1.68$, $k = \frac{\pi}{6}$, $h = 12$*

To compute c, substitute one of the coordinate pairs, (t, y), into the function. Using the pair (1, 10.67) and a calculator will yield $c \approx -1.683257242$.

The function $y = 1.68 \sin\left(\frac{\pi}{6}t - 1.68\right) + 12.21$ is one model for the daylight in Center City.

b. To find the amount of daylight on May 30, use $t = 5.5$ because May 30 is half a month past May 15, which is represented by $t = 5$.

$y = 1.68 \sin\left(\frac{\pi}{6}t - 1.68\right) + 12.21$

$y = 1.68 \sin\left(\frac{\pi}{6}(5.5) - 1.68\right) + 12.21$

$y \approx 13.78$

On May 30, Center City will have about 13.78 hours of daylight.

Example 2

ZOOLOGY In predator-prey situations, the number of animals in each category tends to vary periodically. A certain region has pumas as predators and deer as prey. The number of pumas varies with time according to the function $P = 500 + 200 \sin 0.4(t - 2)$, and the number of deer varies according to the function $D = 1500 + 400 \sin 0.4t$, with time, t, in years. Use the functions to determine how many pumas and deer there will be in the region in 15 years.

Pumas: Evaluate the function $P = 500 + 200 \sin 0.4(t - 2)$ for $t = 15$.

$P = 500 + 200 \sin 0.4(15 - 2)$
$P = 500 + 200 \sin 0.4(13)$
$P = 323.31$

There will be about 323 pumas in the region in 15 years.

Deer: Evaluate the function $D = 1500 + 400 \sin 0.4t$ for $t = 15$.

$D = 1500 + 400 \sin 0.4(15)$
$D = 1388.23$

There will be about 1,388 deer in the region in 15 years.