Lesson 6-8

Example 1

Write the inverse of y = Arctan $4x$. Then graph the function and its inverse.

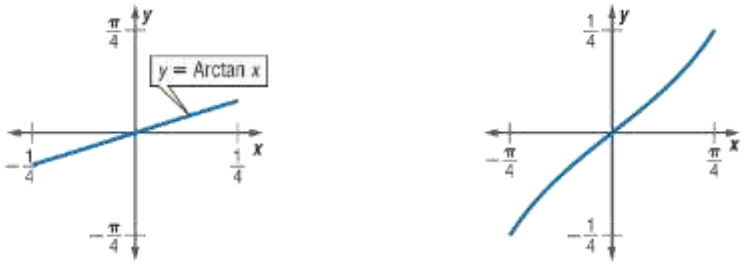
$y = \text{Arctan } 4x$

$x = \text{Arctan } 4y$ *Exchange x and y.*

$\text{Tan } x = 4y$ *Definition of Arctan function*

$\frac{1}{4}\text{Tan } x = y$ *Divide each side by 4.*

Now graph the functions.



Example 2

Find each value.

a. Arcsin (-1)

Let θ = Arcsin (-1).

$\text{Sin } \theta = -1$

$\theta = -\frac{\pi}{2}$

b. Sin^{-1} (cos 2π)

If y = cos 2π, then y = 1.

$\text{Sin}^{-1}(\cos 2\pi) = \text{Sin}^{-1} 1$

$= \frac{\pi}{2}$

c. sin (Tan^{-1} 1 – Sin^{-1} 0)

Let α = Tan^{-1} 1 and β = Sin^{-1} 0.

Tan α = 1 Sin β = 0

$\alpha = \frac{\pi}{4}$ β = 0

sin (Tan^{-1} 1 – Sin^{-1} 0) = sin(α - β)

= sin $1\frac{\pi}{4}$ - 02

$= \frac{\sqrt{2}}{2}$

Example 3

Determine if Sin^{-1} (sin *x*) = *x* is true or false for all values of *x*. If false, give a counterexample.

Try several values of x to see if we can find a counterexample.

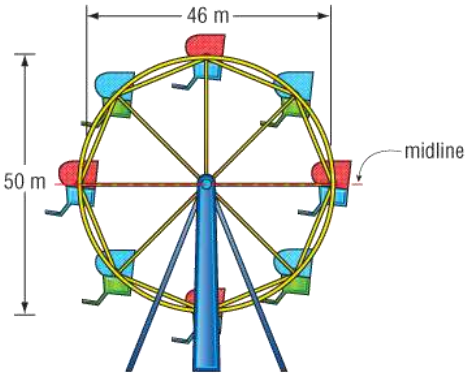
When $x = \pi$, Sin^{-1} (sin *x*) ≠ *x*, so it is not true for all values of *x*.

x	sin *x*	Sin^{-1} (sin *x*)
0	0	0
$\frac{\pi}{2}$	1	$\frac{\pi}{2}$
π	0	0

Example 4

ENTERTAINMENT **A giant Ferris wheel has a height of 50 meters and a diameter of 46 meters. It makes a revolution every 4 minutes. Sue starts timing her ride at the midline point at exactly 10:00 A.M. as she is on her way up. At what time will she reach an altitude of 40 meters?**

First, write an equation to model the height of a seat at any time t. Since the seat is at the midline point at $t = 0$, use the sine function $y = A \sin (kt + c) + h$. Find the values of A, k, c, and h.



***A*:** The value of A is the radius of the Ferris wheel.

$A = \frac{1}{2}(46)$ or 23

***k*:** $\frac{2\pi}{k} = 4$

$k = \frac{\pi}{2}$

***c*:** Since the seat is at the equilibrium point at $t = 0$, there is no phase shift and $c = 0$.

***h*:** The bottom of the Ferris wheel is 50 - 46 or 4 meters above the ground. So, the value of h is 23 + 4 or 27.

Substitute these values into the general equation. The equation is $y = 23 \sin 1\frac{\pi}{2}t2 + 27$.

Now, solve the equation for 40.

$40 = 23 \sin 1\frac{\pi}{2}t2 + 27$		*Replace y with 40.*
$13 = 23 \sin 1\frac{\pi}{2}t2$		*Subtract 27 from each side.*
$\frac{13}{23} = \sin 1\frac{\pi}{2}t2$		*Divide each side by 23.*
$\sin^{-1} 1\frac{13}{23}2 = \frac{\pi}{2}t$		*Definition of $\sin^{-1}$*
$\frac{2}{\pi} \sin^{-1} 1\frac{13}{23}2 = t$		*Multiply each side by $\frac{2}{\pi}$.*
$0.3824154301 = t$		*Use a calculator.*

Sue will reach an altitude of 40 meters about 0.38 minute after 10:00 A.M.