File LibraryLesson 7-2

Example 1

Verify that $\dfrac{\cot x + \cos x \sin x}{\tan x} + \dfrac{\cos x \sin x}{\cot x} = \csc^2 x$ **is an identity.**

Since the left side is more complicated, transform it into the expression on the right.

$$\frac{\cot x + \cos x \sin x}{\tan x} + \frac{\cos x \sin x}{\cot x} \stackrel{?}{=} \csc^2 x$$

$$\frac{\cot x}{\tan x} + \frac{\cos x \sin x}{\tan x} + \frac{\cos x \sin x}{\cot x} \stackrel{?}{=} \csc^2 x$$

$$(\cot x)(\cot x) + (\cos x \sin x)(\cot x) + (\cos x \sin x)(\tan x) \stackrel{?}{=} \csc^2 x \qquad \frac{1}{\tan x} = \cot x,\ \frac{1}{\cot x} = \tan x$$

$$\cot^2 x + (\cos x \sin x)1\frac{\cos x}{\sin x}2 + (\cos x \sin x)1\frac{\sin x}{\cos x}2 \stackrel{?}{=} \csc^2 x \qquad \cot x = \frac{\cos x}{\sin x},\ \tan x = \frac{\sin x}{\cos x}$$

$$\cot^2 x + \cos^2 x + \sin^2 x \stackrel{?}{=} \csc^2 x$$

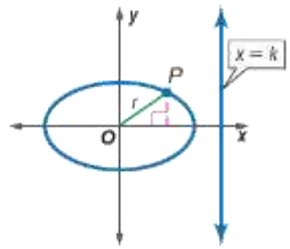
$$\cot^2 x + 1 \stackrel{?}{=} \csc^2 x \qquad \cos^2 x + \sin^2 x = 1$$

$$\csc^2 x = \csc^2 x \qquad \cot^2 x + 1 = \csc^2 x$$

We have transformed the left side into the right side. The identity is verified.

Example 2

PROBLEM SOLVING **In the Cartesian coordinate system, points are expressed as ordered pairs of their *x*- and *y*-coordinates, (*x*, *y*), on a coordinate plane where *x* is the signed horizontal distance from the origin and *y* is the signed vertical distance from the origin. Another way of expressing a point *P* is in polar coordinates, (*r*, *θ*), where *r* is the signed distance of a point from a fixed origin *O*, or pole, and *θ* is the signed angle from the initial ray to the ray *OP*. Using polar coordinates, the equation $r = \frac{ke}{1 + e\cos\theta}$ where $0 < e < 1$, represents an ellipse that has a focus at the origin and directrix $x = k$ to the right of the origin. Show that the equation $r = \frac{ke\sin\theta\csc^2\theta}{\csc\theta + e\cot\theta}$ represents the same ellipse.**



Determine if the statement $\frac{ke\sin\theta\csc^2\theta}{\csc\theta + e\cot\theta} = \frac{ke}{1 + e\cos\theta}$ is an identity.

Begin by writing the left side in terms of sine and cosine.

$$\frac{ke\sin\theta\csc^2\theta}{\csc\theta + e\cot\theta} \stackrel{?}{=} \frac{ke}{1 + e\cos\theta}$$

$$\frac{\frac{ke\sin\theta}{\sin^2\theta}}{\frac{1}{\sin\theta} + e\frac{\cos\theta}{\sin\theta}} \stackrel{?}{=} \frac{ke}{1 + e\cos\theta}$$ *$\csc\theta = \frac{1}{\sin\theta}$, $\cot\theta = \frac{\cos\theta}{\sin\theta}$*

$$\frac{\frac{ke}{\sin\theta}}{\frac{1}{\sin\theta} + e\frac{\cos\theta}{\sin\theta}} \stackrel{?}{=} \frac{ke}{1 + e\cos\theta}$$ *Simplify.*

$$\frac{ke}{1 + e\cos\theta} = \frac{ke}{1 + e\cos\theta}$$ *Multiply the numerator and denominator by sin θ.*

Therefore, $r = \frac{ke\sin\theta\csc^2\theta}{\csc\theta + e\cot\theta}$ does represent the same ellipse as $r = \frac{ke}{1 + e\cos\theta}$.

Example 3

Verify that $\frac{\tan B \sin B}{\sec B} + \cos^2 B = \sec^2 B - \frac{\tan B}{\cot B}$ is an identity.

Since the two sides are equally complicated, we will transform each side independently into the same form.

$$\frac{\tan B \sin B}{\sec B} + \cos^2 B \stackrel{?}{=} \sec^2 B - \frac{\tan B}{\cot B}$$

$$1\frac{\sin B}{\cos B}2(\cos B)(\sin B) + \cos^2 B \stackrel{?}{=} \sec^2 B - (\tan B)(\tan B) \qquad \tan B = \frac{\sin B}{\cos B}, \frac{1}{\sec B} = \cos B, \frac{1}{\cot B} = \tan B$$

$$\sin^2 B + \cos^2 B \stackrel{?}{=} \tan^2 B + 1 - \tan^2 B \qquad \sec^2 B = \tan^2 B + 1$$

$$1 = 1 \qquad \sin^2 B + \cos^2 B = 1$$

Example 4

Find a numerical value of one trigonometric function of x if $\frac{\sin x}{\cot x} + \cos x = 4$.

You can simplify the trigonometric expression on the left side by writing it in terms of sine and cosine.

$$\frac{\sin x}{\cot x} + \cos x = 4$$

$$\frac{\sin x}{\frac{\cos x}{\sin x}} + \cos x = 4 \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\sin x \cdot \frac{\sin x}{\cos x} + \cos x = 4 \qquad \textit{Definition of division}$$

$$\frac{\sin^2 x}{\cos x} + \cos x = 4 \qquad \textit{Simplify.}$$

$$\frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x} = 4$$

$$\frac{\sin^2 x + \cos^2 x}{\cos x} = 4$$

$$\frac{1}{\cos x} = 4 \qquad \sin^2 x + \cos^2 x = 1$$

$$\sec x = 4 \qquad \frac{1}{\cos x} = \sec x$$

Therefore, if $\frac{\sin x}{\cot x} + \cos x = 4$, then $\sec x = 4$.
