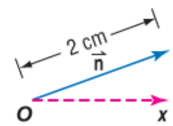
## Lesson 8-1 Geometric Vectors

### Example 1

**Use a ruler and protractor to determine the magnitude (in centimeters) and the direction of $\vec{n}$.**


Sketch the vector in standard position and measure the magnitude and direction. The magnitude is 2 centimeters, and the direction is 20°.

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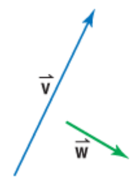
### Example 2

**Find the sum of $\vec{v}$ and $\vec{w}$ using:**




**a.** **the parallelogram method.**

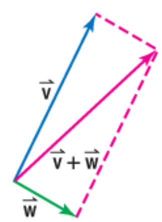
**b.** **the triangle method.**

**c.** **Compare the resultants found in both methods.**

**a.** Copy $\vec{v}$ then copy $\vec{w}$ placing the initial points together.

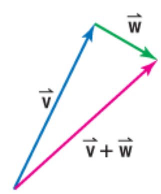
Form a parallelogram that has the two vectors as two of its sides. Draw dashed lines to represent the other two sides.

The resultant is the vector from the vertex of $\vec{v}$ and $\vec{w}$ to the opposite vertex of the parallelogram.




**b.** Copy $\vec{v}$ then copy $\vec{w}$ so that the initial point of $\vec{w}$ is on the terminal point of $\vec{v}$.

The resultant is the vector from the initial point of $\vec{v}$ to the terminal point of $\vec{w}$.




**c.** Use a ruler and protractor to measure the magnitude and direction of each resultant. The resultants of both methods have magnitudes of 2.6 centimeters and directions of 42°. So, the resultants found in both methods are equal.

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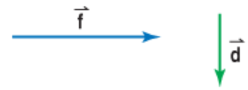
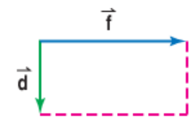
## Example 3

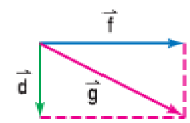
**RECREATION A hang-glider traveled forward at 4 m/s and descended at 2 m/s. Determine the magnitude of the resultant velocity of the hang-glider.**




Let 1 centimeter represent 2 m/s. Draw two vectors, $\vec{\mathbf{f}}$ and $\vec{\mathbf{d}}$, to represent the forward velocity and the descending velocity of the hang-glider, respectively.




Use the parallelogram method. Copy the forward velocity vector. Then copy the descending velocity vector placing the initial point at the initial point of the forward velocity vector. Draw dashed lines to represent the other two sides of the parallelogram.




The resultant velocity of the hang-glider is the vector from the vertex of the two original vectors to the opposite vertex of the parallelogram. Measure the resultant, 2.3 centimeters. Multiply the resultant's magnitude by 2 to determine the magnitude of the resultant velocity of the hang-glider.

2.3 cm ( 2 m/s = 4.6 m/s

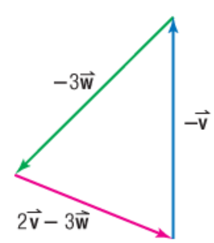
The hang-glider is moving at 4.6 m/s.

## Example 4

**Use the triangle method to find $2\vec{v} - 3\vec{w}$.**


Rewrite the expression as a sum.

$$2\vec{v} - 3\vec{w} = 2\vec{v} + (-3\vec{w})$$




Draw a vector twice the magnitude of $\vec{\mathbf{v}}$ to represent $2\vec{\mathbf{v}}$. Draw a vector with the opposite direction to $\vec{\mathbf{w}}$ and three times its magnitude to represent $3\vec{\mathbf{w}}$.

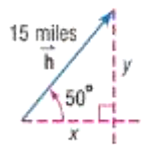
Place the initial point of the second vector on the terminal point of the first vector.
*(Tip-to-tail method)*

The resultant vector is the answer.

## Example 5

**A hiker leaves her camp and walks for 15 miles in a direction 50° north of due east. Find the magnitude of her vertical and horizontal components.**

Draw $\vec{\mathbf{h}}$. Then draw a horizontal vector through the initial point of $\vec{\mathbf{h}}$. Label the resulting angle 50°. Draw a vertical vector through the terminal point of $\vec{\mathbf{h}}$. The vectors will form a right triangle, so you can use the sine and cosine ratios to find the magnitude of the components.




$$\sin 50° = \frac{y}{15} \qquad \cos 50° = \frac{x}{15}$$

$$y = 15 \sin 50° \qquad x = 15 \cos 50°$$

$$y \approx 11 \qquad x \approx 10$$

The magnitude of the vertical component is approximately 11 miles, and the magnitude of the horizontal component is approximately 10 miles.

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## Example 6

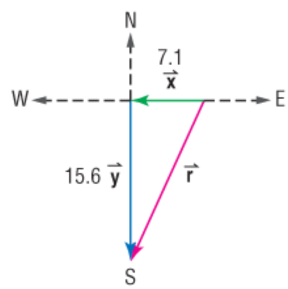
**During a windy soccer game, the ball is kicked with a velocity of 15.6 miles per hour in a southerly direction. A gust of wind carries the ball with an additional velocity of 7.1 miles per hour in a westerly direction.**

**a. What is the magnitude of the resultant velocity of the ball, to the nearest tenth of a mile per hour?**

**b. What is the direction of the resultant velocity of the ball, to the nearest degree?**

**a.** Let $\vec{\mathbf{y}}$ represent the velocity due to the kick.

Let $\vec{\mathbf{x}}$ represent the velocity due to the wind.




Draw the resultant $\vec{\mathbf{r}}$. This represents the total velocity of the ball. Use the Pythagorean Theorem to find the magnitude of the resultant.

$$c^2 = a^2 + b^2$$

$$\vec{\mathbf{r}}^{\,2} = 15.6^2 + 7.1^2$$

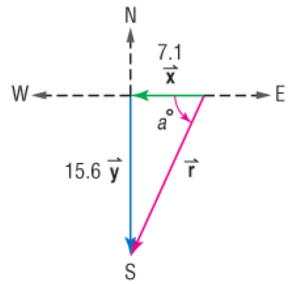
$$|\vec{\mathbf{r}}| = \sqrt{(15.6)^2 + (7.1)^2}$$

$$|\vec{\mathbf{r}}| \approx 17.1$$

The magnitude of the resultant velocity of the ball is about 17.1 miles per hour.

**b.** Let $a$ be the measure of the angle $\vec{\mathbf{r}}$ makes with $\vec{\mathbf{x}}$.

The direction of the resultant can be found by using the tangent ratio.




$$\tan a = \frac{|\vec{\mathbf{y}}|}{|\vec{\mathbf{x}}|}$$

$$\tan a = \frac{15.6}{7.1}$$

$$a \approx 66°$$ *Take $\tan^{-1}$ of each side.*

The resultant makes an angle of 66° with $\vec{\mathbf{x}}$. The direction of the resultant velocity upon the ball is 90 - 66 or 24° west of south.