Lesson 8-4 Perpendicular Vectors

Example 1

Find each inner product if $\vec{p} = \langle 3, 7\rangle$, $\vec{q} = \langle 2, -3\rangle$, and $\vec{m} = \langle 9, 6\rangle$. Are any pairs of vectors perpendicular?

a. $\vec{p} \cdot \vec{q}$

$\vec{p} \cdot \vec{q} = 3(2) + 7(-3)$
$= 6 - 21$
$= -15$

$\vec{p}$ and $\vec{q}$ are not perpendicular.

b. $\vec{p} \cdot \vec{m}$

$\vec{p} \cdot \vec{m} = 3(9) + 7(6)$
$= 27 + 42$
$= 69$

$\vec{p}$ and $\vec{m}$ are not perpendicular.

c. $\vec{q} \cdot \vec{m}$

$\vec{q} \cdot \vec{m} = 2(9) + (-3)(6)$
$= 18 - 18$
$= 0$

$\vec{q}$ and $\vec{m}$ are perpendicular.

Example 2

Find the inner product of $\vec{a}$ and $\vec{b}$ if $\vec{a} = \langle -4, 2, 5\rangle$ and $\vec{b} = \langle 3, 6, 1\rangle$. Are the two vectors perpendicular?

$\vec{a} \cdot \vec{b} = (-4)(3) + (2)(6) + (5)(1)$
$= -12 + 12 + 5$
$= 5$

The two vectors are not perpendicular since their inner product is not zero.

Example 3

Find the cross product of $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ if $\vec{\mathbf{v}}$ = ⟨5, 2, 3⟩ and $\vec{\mathbf{w}}$ = ⟨-2, 5, 0⟩. Verify that the resulting vector is perpendicular to the two original vectors.

$$\vec{\mathbf{v}} \,(\, \vec{\mathbf{w}} = \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 5 & 2 & 3 \\ -2 & 5 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 \\ 5 & 0 \end{vmatrix} \vec{\mathbf{i}} - \begin{vmatrix} 5 & 3 \\ -2 & 0 \end{vmatrix} \vec{\mathbf{j}} + \begin{vmatrix} 5 & 2 \\ -2 & 5 \end{vmatrix} \vec{\mathbf{k}}$$ *Expand by minors.*

$$= -15\vec{\mathbf{i}} - 6\vec{\mathbf{j}} + 29\vec{\mathbf{k}} \text{ or } \langle -15, -6, 29 \rangle$$

Find the inner products ⟨-15, -6, 29⟩ · ⟨5, 2, 3⟩ and ⟨-15, -6, 29⟩ · ⟨-2, 5, 0⟩.

$-15(5) + (-6)(2) + (29)(3) = 0$

$(-15)(-2) + (-6)(5) + (29)(0) = 0$

Since the inner products are zero, the cross product $\vec{\mathbf{v}}$ ($\vec{\mathbf{w}}$ (-15, -6, 29) is perpendicular to both $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$.

Example 4

AUTO RACING Suppose Graham is applying a force of 20 pounds along the *z*-axis to the gearshift of his car. If the center of the connection of the gearshift is at the origin, the force is applied at the point (0.6, 0, 0.3). Find the torque.

We need to find $|\vec{\mathbf{T}}|$, the torque of the force at (0.6, 0, 0.3) where each value is the distance from the origin in feet and $\vec{\mathbf{F}}$ represents the force in pounds.

To find the magnitude of $\vec{\mathbf{T}}$, we must first find $\overrightarrow{AB}$ and $\vec{\mathbf{F}}$.

$$\begin{aligned}\overrightarrow{AB} &= (0.6, 0, 0.3) - (0, 0, 0)\\ &= \langle 0.6 - 0, 0 - 0, 0.3 - 0\rangle \text{ or } \langle 0.6, 0, 0.3\rangle\end{aligned}$$

Any upward force is measured along the *z*-axis, so $\vec{\mathbf{F}} = 20\vec{\mathbf{k}}$ or $\langle 0, 0, 20\rangle$.

Now, find $\vec{\mathbf{T}}$.

$$\begin{aligned}\vec{\mathbf{T}} &= \overrightarrow{AB} \; (\; \vec{\mathbf{F}}\\ &= \begin{vmatrix} \vec{\mathbf{i}} & \vec{\mathbf{j}} & \vec{\mathbf{k}} \\ 0.6 & 0 & 0.3 \\ 0 & 0 & 20 \end{vmatrix}\\ &= \begin{vmatrix} 0 & 0.3 \\ 0 & 20 \end{vmatrix}\vec{\mathbf{i}} - \begin{vmatrix} 0.6 & 0.3 \\ 0 & 20 \end{vmatrix}\vec{\mathbf{j}} + \begin{vmatrix} 0.6 & 0 \\ 0 & 0 \end{vmatrix}\vec{\mathbf{k}} \qquad \textit{Expand by minors.}\\ &= 0\vec{\mathbf{i}} - 12\vec{\mathbf{j}} + 0\vec{\mathbf{k}} \text{ or } \langle 0, -12, 0\rangle\end{aligned}$$

Find the magnitude of $\vec{\mathbf{T}}$.

$$\begin{aligned}|\vec{\mathbf{T}}| &= \sqrt{0^2 + (-12)^2 + 0^2}\\ &= \sqrt{(-12)^2} \text{ or } 12\end{aligned}$$

The torque is about 12 foot-pounds.