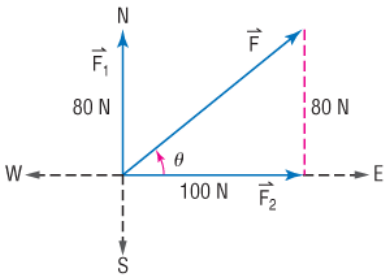
Lesson 8-5 Applications With Vectors

Example 1

DOGS **Two children are attempting to capture a loose dog. One of the children is exerting a force of 80 Newtons due north and the other is pulling with a force of 100 Newtons due east. What is the resultant force on the dog?**

a. Draw a labeled diagram that represents the forces.

Let $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ represent the forces exerted by the children. Then $\vec{\mathbf{F}}$ represents the resultant. Let θ represent the angle $\vec{\mathbf{F}}$ makes with the east-west or x-axis.



b. Determine the resultant force exerted on the dog by the two children.

$$|\vec{\mathbf{F}}|^2 = |\vec{\mathbf{F}}_1|^2 + |\vec{\mathbf{F}}_2|^2$$
$$|\vec{\mathbf{F}}|^2 = (80)^2 + (100)^2$$
$$|\vec{\mathbf{F}}|^2 = 16{,}400$$
$$\sqrt{|\vec{\mathbf{F}}|^2} = \sqrt{16{,}400} \text{ or about } 128.1$$

The resultant force on the dog is about 128.1 Newtons. *1 pound is about 4.45 Newtons, so 128.1 N is about 28.8 pounds.*

c. Find the angle the resultant force makes with the east-west axis.

Use the tangent ratio.

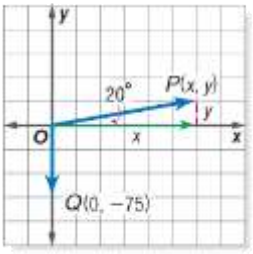
$$\tan\theta = \frac{80}{100}$$
$$\theta = \tan^{-1}\frac{80}{100}$$
$$\theta \approx 38.7° \text{ north of due east}$$

The resultant force is applied at an angle of 38.7° north of due east.

Example 2

Sam works for a grocery store. Suppose he is pushing a cart full of produce weighing 75 pounds up a ramp 10 feet long at an incline of 20°. Find the work done by gravity as the cart moves the length of the ramp. Assume that friction is not a factor.

First draw a labeled diagram representing the forces involved. Let $\overrightarrow{OQ}$ represent the force of gravity, or weight. The weight has a magnitude of 75 pounds and its direction is down. The corresponding unit vector is $0\,\vec{\mathbf{i}} - 75\,\vec{\mathbf{j}}$. So, $\vec{\mathbf{F}} = 0\,\vec{\mathbf{i}} - 75\,\vec{\mathbf{j}}$. The application of the force is $\overrightarrow{OP}$, and it has a magnitude of 10 feet.



Write $\overrightarrow{OP}$ as and use trigonometry to find *x* and *y*.

$$\cos 20° = \frac{x}{10} \qquad \sin 20° = \frac{y}{10}$$
$$x = 10 \cos 20° \qquad y = 10 \sin 20°$$
$$x \approx 9.40 \qquad y \approx 3.42$$

Then, $\vec{\mathbf{d}} = 9.40\,\vec{\mathbf{i}} + 3.42\,\vec{\mathbf{j}}$.

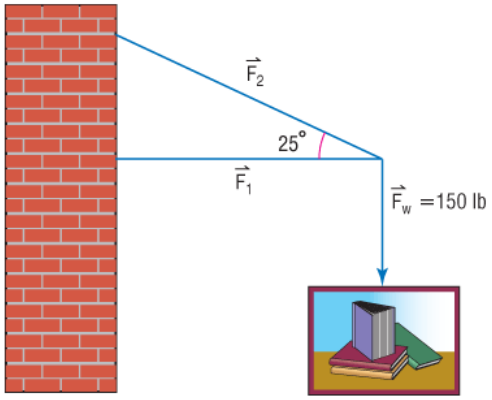
Apply the formula for determining the work done by gravity.

$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}}$
$W = \langle 0\,\vec{\mathbf{i}} - 75\,\vec{\mathbf{j}} \rangle \cdot \langle 9.40\,\vec{\mathbf{i}} + 3.42\,\vec{\mathbf{j}} \rangle$
$W = 0 - 256.5$ or -256.5

Work done by gravity is negative when an object is lifted or raised. As the cart moves the length of the ramp, the work done by gravity is –256.5 ft-lb.

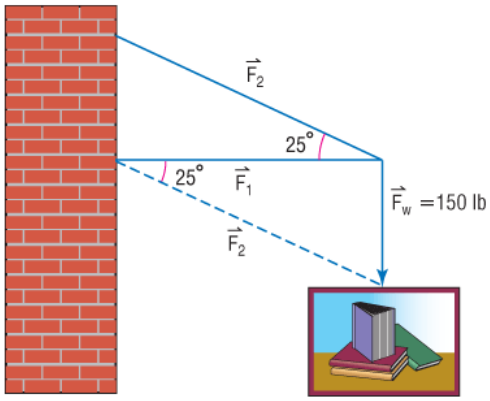
Example 3

Danielle is hanging a sign for her new book store. The sign is supported by two lightweight support bars as shown in the diagrams. If the bars make a 25° angle with each other, and the sign weighs 150 pounds, what are the magnitudes of the forces exerted by the sign on each support bar?



$\vec{F}_1$ represents the force exerted on bar 1 by the sign, $\vec{F}_2$ represents the force exerted on bar 2 by the sign, and $\vec{F_w}$ represents the weight of the sign.

Remember that equal vectors have the same magnitude and direction. So by drawing another vector from the initial point of $\vec{F}_1$ to the terminal point of $\vec{F_w}$, we can use the sine and cosine ratios to determine $|\vec{F}_1|$ and $|\vec{F}_2|$.



$$\sin 25° = \frac{150}{|\vec{F}_2|}$$

$$|\vec{F}_2| = \frac{150}{\sin 25°}$$

$$\approx 354.93$$

$$\cos 25° = \frac{|\vec{F}_1|}{354.93}$$

$$|\vec{F}_1| = 354.93 \cos 25°$$

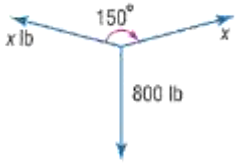
$$\approx 321.68$$

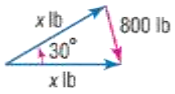
The sign exerts a force of about 322 pounds on bar 1 and a force of about 355 pounds on bar 2.

Example 4

A lighting system for a restaurant is supported equally by two cables suspended from the ceiling of the restaurant. The cables form a 150° angle with each other. If the lighting system weighs 800 pounds, what is the force exerted by each of the cables on the lighting system?

Draw a diagram of the situation. Then draw the vectors tip-to-tail.





Since the triangle is isosceles, the base angles are congruent. Thus, each base angle measures $\frac{180° - 30°}{2}$ or 75°. We can use the Law of Sines to find the force exerted by the cables.

$$\frac{800}{\sin 30°} = \frac{x}{\sin 75°} \qquad \textit{Law of Sines}$$

$$x = \frac{800 \sin 75°}{\sin 30°}$$

$$x \approx 1545.48$$

The force exerted by each cable is about 1545 pounds.