Lesson 8-6 Vectors and Parametric Equations

Example 1

Write a vector equation describing a line passing through $P_1(3, 2)$ and parallel to $\vec{\mathbf{a}} = \langle 4, -1 \rangle$.

Let the line ℓ through $P_1(3, 2)$ be parallel to $\vec{\mathbf{a}}$. For any point $P(x, y)$ on ℓ, $\overrightarrow{P_1P} = \langle x - 3, y - 2 \rangle$. Since P_1P is on ℓ and is parallel to $\vec{\mathbf{a}}$, $\overrightarrow{P_1P} = t\vec{\mathbf{a}}$ for some value t. By substitution, we have $\langle x - 3, y - 2 \rangle = t\langle 4, -1 \rangle$.

Therefore, the equation $\langle x - 3, y - 2 \rangle = t\langle 4, -1 \rangle$ is a vector equation describing all of the points (x, y) on ℓ parallel to $\vec{\mathbf{a}}$ through $P_1(3, 2)$.

Example 2

Find the parametric equations for a line parallel to $\vec{\mathbf{q}} = \langle 4, -2 \rangle$ and passing through the point at (-1, -3). Then make a table of values and graph the line.

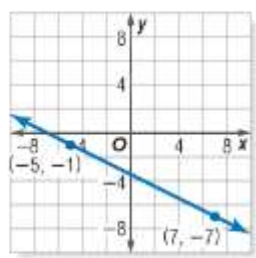
Use the general form of the parametric equations of a line with $\langle a_1, a_2 \rangle = \langle 4, -2 \rangle$ and $\langle x_1, y_1 \rangle = \langle -1, -3 \rangle$.

$x = x_1 + ta_1$
$x = -1 + t(4)$
$x = -1 + 4t$

$y = y_1 + ta_2$
$y = -3 + t(-2)$
$y = -3 - 2t$

Now make a table of values for t. Evaluate each expression to find values for x and y.
Then graph the line.

t	*x*	*y*
-1	-5	-1
0	-1	-3
1	3	-5
2	7	-7

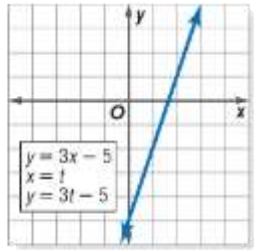


Example 3

Write parametric equations of $y = 3x - 5$.

In the equation $y = 3x - 5$, x is the independent variable, and y is the dependent variable. In parametric equations, t is the independent variable, and x and y are dependent variables. If we set the independent variables x and t equal, we can write two parametric equations in terms of t.

$x = t$
$y = 3t - 5$



Parametric equations for the line are $x = t$ and $y = 3t - 5$.

By making a table of values for t and evaluating each expression to find values of x and y and graphing the line, the parametric equations describe the same line as $y = 3x - 5$.

Example 4

CAR RACING Car #1 leaves the starting line at a speed of 60 mph. Two minutes later, car #2 leaves the starting line at a speed of 64 mph. They are racing for the next 400 miles. Use parametric equations to model the situation. Assume that both cars maintain a constant speed.

a. How long is it until car #2 overtakes car #1?

b. How far have both cars traveled when the second car overtakes the first car?

a. First, write a set of parametric equations to represent each car's position at t hours.
Car #1: $x = 60t$ *$x = vt$*
Car #2: $x = 64(t - 0.0333)$ *2 minutes ≈ 0.0333 h*

Since the time at which the second car overtakes the first is when they have traveled the same distance, set the two expressions equal to each other.

$$60t \approx 64(t - 0.0333)$$
$$60t \approx 64t - 2.1312$$
$$2.1312 \approx 4t$$
$$0.5328 \approx t$$

In about 0.5328 hour or 32 minutes, the second car overtakes the first.

b. Use the time to find the distance traveled when the cars pass.
$x = 60t$
$x = 60(0.5328)$
$x = 31.968$ or about 32 miles

The cars have traveled about 32 miles when the second car overtakes the first car.

Example 5

Write an equation in slope-intercept form of the line whose parametric equations are $x = 3 + 2t$ and $y = -1 - 4t$.

Solve each parametric equation for t.

$$x = 3 + 2t$$
$$x - 3 = 2t$$
$$\frac{x - 3}{2} = t$$

$$y = -1 - 4t$$
$$y + 1 = -4t$$
$$\frac{-1 - y}{4} = t$$

Use substitution to write an equation for the line without the variable t.

$$\frac{x - 3}{2} = \frac{-1 - y}{4} \quad \textit{Substitution}$$
$$4(x - 3) = 2(-1 - y) \quad \textit{Cross multiply.}$$
$$4x - 12 = -2 - 2y \quad \textit{Simplify.}$$
$$y = -2x + 5 \quad \textit{Solve for } y.$$