

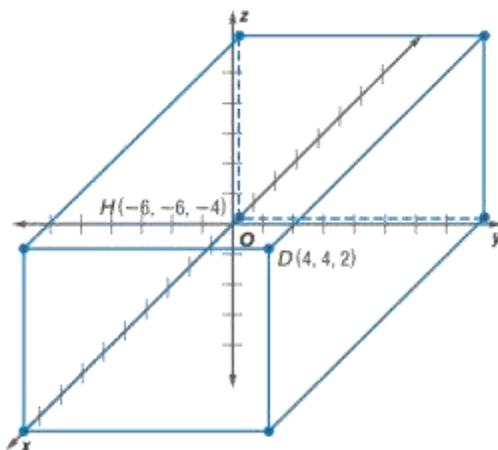
Lesson 8-8 Transformational Matrices in Three-Dimensional Space

Example 1

The vertices of a rectangular prism are given by $A(4, 4, -4)$, $B(4, -4, -4)$, $C(4, -4, 2)$, $D(4, 4, 2)$, $E(-6, -4, 2)$, $F(-6, 4, 2)$, $G(-6, 4, -4)$, and $H(-6, -4, -4)$. Represent these vertices in a vertex matrix. Then graph the rectangular prism.

The vertex matrix for the prism is

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>
<i>x</i>	4	4	4	4	-6	-6	-6	-6
<i>y</i>	4	-4	-4	4	-4	4	4	-4
<i>z</i>	-4	-4	2	2	2	2	-4	-4



Example 2

Bob needs to translate a prism using the vector $\vec{a} = \langle 2, 2, 0 \rangle$. The vertices of the prism have the following coordinates.

$$A(3, 2, -2) \quad B(0, 0, -3) \quad C(-1, 4, -3)$$

$$D(-1, 4, 4) \quad E(3, 2, 4) \quad F(0, 0, 4)$$

- a. Write a matrix that will have such an effect on the figure.
- b. Find the coordinates of the vertices of the translated image.

a. To translate the prism by the vector $\vec{a} = \langle 2, 2, 0 \rangle$, we must first add 2 to each of the x - and y -coordinates. The z -coordinates remain the same. The translation matrix can be written as

$$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

b. Write the vertices of the prism in a 6×3 matrix. Then add it to the translation matrix to find the vertices of the translated image.

$$\begin{array}{l} \text{Original Matrix} \\ \begin{bmatrix} 3 & 0 & -1 & -1 & 3 & 0 \\ 2 & 0 & 4 & 4 & 2 & 0 \\ -2 & -3 & -3 & 4 & 4 & 4 \end{bmatrix} \end{array} + \begin{array}{l} \text{Translation Matrix} \\ \begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} = \begin{array}{l} \text{Translated Image Matrix} \\ \begin{bmatrix} 5 & 2 & 1 & 1 & 5 & 2 \\ 4 & 2 & 6 & 6 & 4 & 2 \\ -2 & -3 & -3 & 4 & 4 & 4 \end{bmatrix} \end{array}$$

Example 3

Let M represent the vertex matrix of the rectangular prism in Example 1.

a. Find TM if $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$.

b. Graph the resulting image.

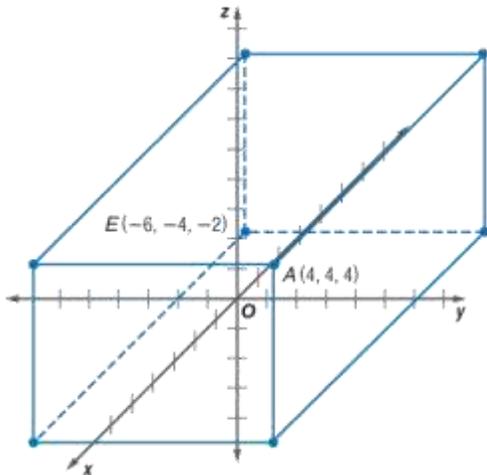
c. Describe the transformation represented by matrix T .

a. First find TM .

$$TM = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 4 & 4 & 4 & -6 & -6 & -6 & -6 \\ 4 & -4 & -4 & 4 & -4 & 4 & 4 & -4 \\ -4 & -4 & 2 & 2 & 2 & 2 & -4 & -4 \end{bmatrix}$$

$$TM = \begin{bmatrix} 4 & 4 & 4 & 4 & -6 & -6 & -6 & -6 \\ 4 & -4 & -4 & 4 & -4 & 4 & 4 & -4 \\ 4 & 4 & -2 & -2 & -2 & -2 & 4 & 4 \end{bmatrix}$$

b. Then graph the points represented by the resulting matrix.



c. The transformation matrix T reflects the image of each vertex over the xy -plane. This results in a reflection of the prism when the new vertices are connected by segments.

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