Name _____

CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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1. If $y = x\sin x$, then $\frac{dy}{dx} =$

   (A) $\sin x + \cos x$

   (B) $\sin x + x\cos x$

   (C) $\sin x - x\cos x$

   (D) $x(\sin x + \cos x)$

   (E) $x(\sin x - \cos x)$

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2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

   (A) $(-\infty, -10]$ and $[10, \infty)$

   (B) $[-10, 10]$

   (C) $[0, 10]$ only

   (D) $[0, 10\sqrt{3}]$ only

   (E) $[0, \infty)$

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3. $\int \sec x \tan x \, dx =$

(A) $\sec x + C$

(B) $\tan x + C$

(C) $\frac{\sec^2 x}{2} + C$

(D) $\frac{\tan^2 x}{2} + C$

(E) $\frac{\sec^2 x \tan^2 x}{2} + C$

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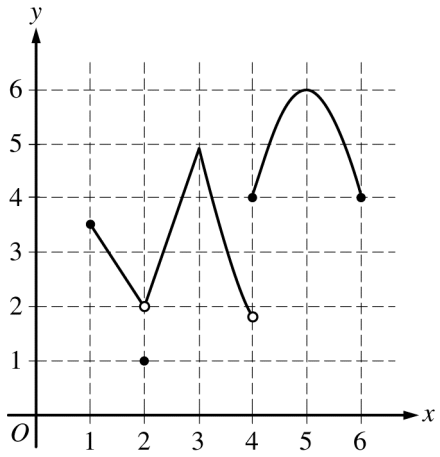
4. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8



Graph of f

5. The graph of the function f is shown above. Which of the following statements is false?

   (A) $\lim_{x \to 2} f(x)$ exists.

   (B) $\lim_{x \to 3} f(x)$ exists.

   (C) $\lim_{x \to 4} f(x)$ exists.

   (D) $\lim_{x \to 5} f(x)$ exists.

   (E) The function f is continuous at $x = 3$.

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6. A particle moves along the x-axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

   (A) 3 (B) 6 (C) 9 (D) 18 (E) 27

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7. If $y = \left(x^3 - \cos x\right)^5$, then $y' =$

(A) $5\left(x^3 - \cos x\right)^4$

(B) $5\left(3x^2 + \sin x\right)^4$

(C) $5\left(3x^2 + \sin x\right)$

(D) $5\left(3x^2 + \sin x\right)^4 \cdot (6x + \cos x)$

(E) $5\left(x^3 - \cos x\right)^4 \cdot \left(3x^2 + \sin x\right)$

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t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

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$$f(x) = \begin{cases} \dfrac{(2x+1)(x-2)}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

9. Let f be the function defined above. For what value of k is f continuous at $x = 2$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

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10. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

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12. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ is equal to which of the following?

(A) $2\int_1^{16} e^u\,du$ (B) $2\int_1^4 e^u\,du$ (C) $2\int_1^2 e^u\,du$ (D) $\frac{1}{2}\int_1^2 e^u\,du$ (E) $\int_1^4 e^u\,du$

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13. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x)\,dx$?

(A) 2 (B) 6 (C) 8 (D) 10 (E) 12

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14. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is

(A) $\frac{7}{\sqrt{5}}$ (B) $\frac{14}{\sqrt{5}}$ (C) $\frac{18}{\sqrt{5}}$ (D) $\frac{15}{\sqrt{21}}$ (E) $\frac{30}{\sqrt{21}}$

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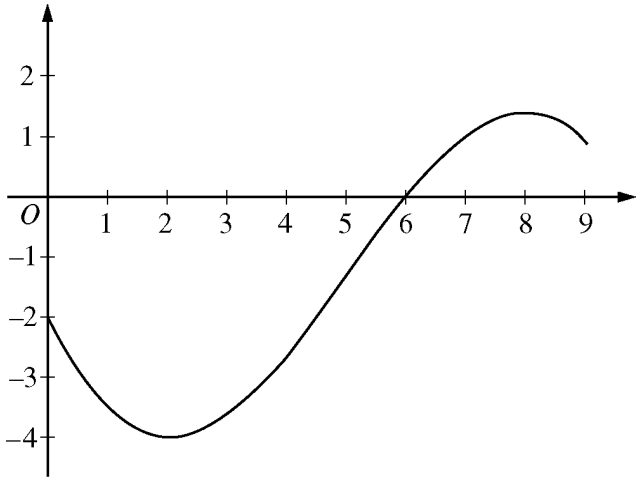


Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

16. A particle moves along the x-axis with its position at time t given by $x(t) = (t - a)(t - b)$, where a and b are constants and $a \neq b$. For which of the following values of t is the particle at rest?

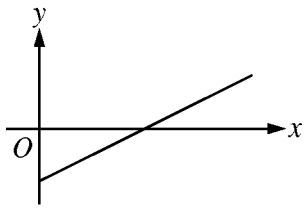
(A) $t = ab$

(B) $t = \dfrac{a + b}{2}$

(C) $t = a + b$

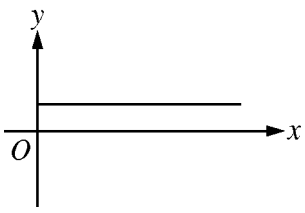
(D) $t = 2(a + b)$

(E) $t = a$ and $t = b$



Graph of f

17. The figure above shows the graph of f. If $f(x) = \int_2^x g(t)\,dt$, which of the following could be the graph of $y = g(x)$?

(A)


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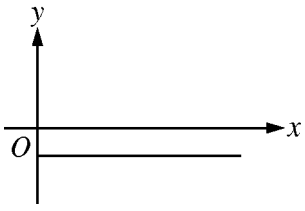

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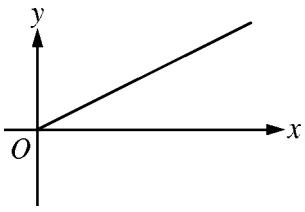

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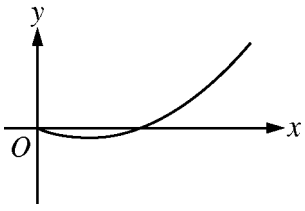

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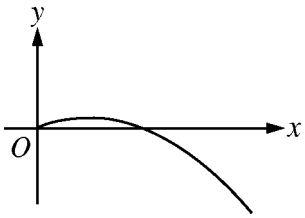

18. $\lim_{h \to 0} \dfrac{\ln(4+h) - \ln(4)}{h}$ is

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) e (E) nonexistent

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19. The function f is defined by $f(x) = \dfrac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\frac{1}{2}, \frac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4, \frac{2}{3}\right)$

(E) There are no such points.

20. Let $f(x) = (2x+1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

(A) $-\frac{2}{27}$ (B) $\frac{1}{54}$ (C) $\frac{1}{27}$ (D) $\frac{1}{6}$ (E) 6

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21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \frac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \frac{1}{x-5}$ (D) $y = \frac{5x}{1-x}$ (E) $y = \frac{20x^2 - x}{1+4x^2}$

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22. Let f be the function defined by $f(x) = \dfrac{\ln x}{x}$. What is the absolute maximum value of f?

(A) 1

(B) $\dfrac{1}{e}$

(C) 0

(D) $-e$

(E) f does not have an absolute maximum value.

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23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\dfrac{dP}{dt} = 200$

(B) $\dfrac{dP}{dt} = 200t$

(C) $\dfrac{dP}{dt} = 100t^2$

(D) $\dfrac{dP}{dt} = 200P$

(E) $\dfrac{dP}{dt} = 100P^2$

24. Let g be the function given by $g(x) = x^2 e^{kx}$, where k is a constant. For what value of k does g have a critical point at $x = \frac{2}{3}$?

(A) -3 (B) $-\frac{3}{2}$ (C) $-\frac{1}{3}$ (D) 0 (E) There is no such k.

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 2\sin x$ with the initial condition $y(\pi) = 1$?

(A) $y = 2\cos x + 3$

(B) $y = 2\cos x - 1$

(C) $y = -2\cos x + 3$

(D) $y = -2\cos x + 1$

(E) $y = -2\cos x - 1$

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26. Let g be a function with first derivative given by $g'(x) = \int_0^x e^{-t^2}\,dt$. Which of the following must be true on the interval $0 < x < 2$?

(A) g is increasing, and the graph of g is concave up.

(B) g is increasing, and the graph of g is concave down.

(C) g is decreasing, and the graph of g is concave up.

(D) g is decreasing, and the graph of g is concave down.

(E) g is decreasing, and the graph of g has a point of inflection on $0 < x < 2$.

27. If $(x + 2y) \cdot \frac{dy}{dx} = 2x - y$, what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$?

(A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$ (E) Undefined

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28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.