Name _____

CALCULUS BC
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. If $y = \sin^3 x$, then $\frac{dy}{dx} =$

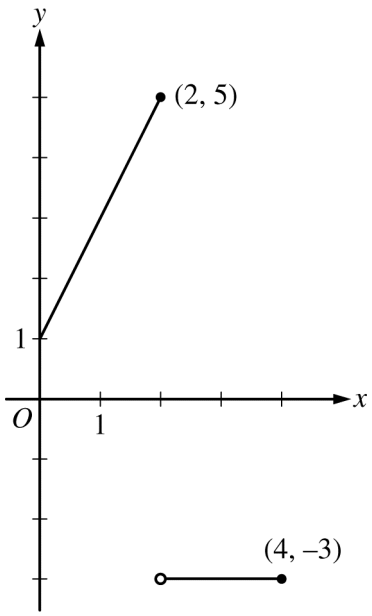
(A) $\cos^3 x$ (B) $3\cos^2 x$ (C) $3\sin^2 x$ (D) $-3\sin^2 x \cos x$ (E) $3\sin^2 x \cos x$

---

2. The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = t^3 - 3t^2$ and $y(t) = 12t - 3t^2$. At which of the following points (x, y) is the particle at rest?

(A) $(-4, 12)$ (B) $(-3, 6)$ (C) $(-2, 9)$ (D) $(0, 0)$ (E) $(3, 4)$

GO ON TO THE NEXT PAGE.



Graph of f

3. The graph of f is shown above for $0 \le x \le 4$. What is the value of $\int_0^4 f(x)\,dx$?

(A) -1 (B) 0 (C) 2 (D) 6 (E) 12

GO ON TO THE NEXT PAGE.

4. Which of the following integrals gives the length of the curve $y = \ln x$ from $x = 1$ to $x = 2$?

(A) $\int_1^2 \sqrt{1 + \frac{1}{x^2}}\, dx$

(B) $\int_1^2 \left(1 + \frac{1}{x^2}\right) dx$

(C) $\int_1^2 \sqrt{1 + e^{2x}}\, dx$

(D) $\int_1^2 \sqrt{1 + (\ln x)^2}\, dx$

(E) $\int_1^2 \left(1 + (\ln x)^2\right) dx$

---

5. The Maclaurin series for the function f is given by $f(x) = \sum_{n=0}^{\infty} \left(-\frac{x}{4}\right)^n$. What is the value of $f(3)$?

(A) -3 (B) $-\frac{3}{7}$ (C) $\frac{4}{7}$ (D) $\frac{13}{16}$ (E) 4

GO ON TO THE NEXT PAGE.

6. Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x\left(x^2 - 3\right)^5 \, dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5 \, du$

(B) $\int_{-2}^{13} u^5 \, du$

(C) $\frac{1}{2}\int_{-2}^{13} u^5 \, du$

(D) $\int_{-1}^{4} u^5 \, du$

(E) $\frac{1}{2}\int_{-1}^{4} u^5 \, du$

---

7. If $\arcsin x = \ln y$, then $\frac{dy}{dx} =$

(A) $\frac{y}{\sqrt{1 - x^2}}$

(B) $\frac{xy}{\sqrt{1 - x^2}}$

(C) $\frac{y}{1 + x^2}$

(D) $e^{\arcsin x}$

(E) $\frac{e^{\arcsin x}}{1 + x^2}$

GO ON TO THE NEXT PAGE.

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

---

9. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \frac{8^n}{n!}$ II. $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$ III. $\sum_{n=1}^{\infty} \frac{n+1}{(n)(n+2)(n+3)}$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

GO ON TO THE NEXT PAGE.

10. $\int_1^4 t^{-3/2}\,dt =$

(A) -1 (B) $-\frac{7}{8}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) 1

---

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

GO ON TO THE NEXT PAGE.

12. The points $(-1, -1)$ and $(1, -5)$ are on the graph of a function $y = f(x)$ that satisfies the differential equation $\frac{dy}{dx} = x^2 + y$. Which of the following must be true?

(A) $(1, -5)$ is a local maximum of f.

(B) $(1, -5)$ is a point of inflection of the graph of f.

(C) $(-1, -1)$ is a local maximum of f.

(D) $(-1, -1)$ is a local minimum of f.

(E) $(-1, -1)$ is a point of inflection of the graph of f.

---

13. What is the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(x-4)^{2n}}{3^n}$?

(A) $2\sqrt{3}$ (B) 3 (C) $\sqrt{3}$ (D) $\frac{\sqrt{3}}{2}$ (E) 0

14. Let k be a positive constant. Which of the following is a logistic differential equation?

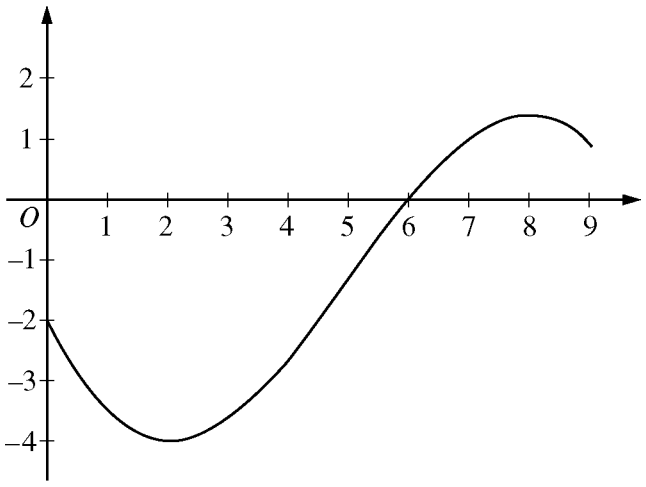
(A) $\frac{dy}{dt} = kt$

(B) $\frac{dy}{dt} = ky$

(C) $\frac{dy}{dt} = kt(1 - t)$

(D) $\frac{dy}{dt} = ky(1 - t)$

(E) $\frac{dy}{dt} = ky(1 - y)$



Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

GO ON TO THE NEXT PAGE.

16. Let $y = f(x)$ be the solution to the differential equation $\frac{dy}{dx} = x - y$ with initial condition $f(1) = 3$. What is the approximation for $f(2)$ obtained by using Euler's method with two steps of equal length starting at $x = 1$?

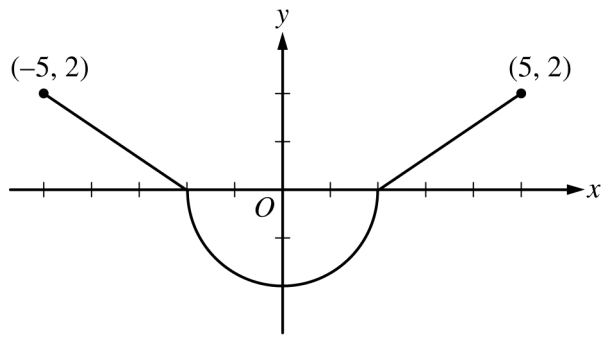
(A) $-\frac{5}{4}$ (B) 1 (C) $\frac{7}{4}$ (D) 2 (E) $\frac{21}{4}$

---

17. For $x > 0$, the power series $1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \frac{x^6}{7!} + \cdots + (-1)^n \frac{x^{2n}}{(2n+1)!} + \cdots$ converges to which of the following?

(A) $\cos x$ (B) $\sin x$ (C) $\frac{\sin x}{x}$ (D) $e^x - e^{x^2}$ (E) $1 + e^x - e^{x^2}$

GO ON TO THE NEXT PAGE.



Graph of f'

18. The graph of f', the derivative of a function f, consists of two line segments and a semicircle, as shown in the figure above. If $f(2) = 1$, then $f(-5) =$

(A) $2\pi - 2$

(B) $2\pi - 3$

(C) $2\pi - 5$

(D) $6 - 2\pi$

(E) $4 - 2\pi$

19. The function f is defined by $f(x) = \dfrac{x}{x+2}$. What points (x,y) on the graph of f have the property that the line tangent to f at (x,y) has slope $\dfrac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\dfrac{1}{2},\dfrac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4,\dfrac{2}{3}\right)$

(E) There are no such points.

---

20. $\displaystyle\int_0^1 \frac{5x+8}{x^2+3x+2}\,dx$ is

(A) $\ln(8)$ (B) $\ln\left(\dfrac{27}{2}\right)$ (C) $\ln(18)$ (D) $\ln(288)$ (E) divergent

GO ON TO THE NEXT PAGE.

21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \dfrac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \dfrac{1}{x-5}$ (D) $y = \dfrac{5x}{1-x}$ (E) $y = \dfrac{20x^2 - x}{1+4x^2}$

---

22. The power series $\sum_{n=0}^{\infty} a_n (x-3)^n$ converges at $x = 5$. Which of the following must be true?

(A) The series diverges at $x = 0$.

(B) The series diverges at $x = 1$.

(C) The series converges at $x = 1$.

(D) The series converges at $x = 2$.

(E) The series converges at $x = 6$.

23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\frac{dP}{dt} = 200$

(B) $\frac{dP}{dt} = 200t$

(C) $\frac{dP}{dt} = 100t^2$

(D) $\frac{dP}{dt} = 200P$

(E) $\frac{dP}{dt} = 100P^2$

---

24. Let f be a differentiable function such that $\int f(x)\sin x\,dx = -f(x)\cos x + \int 4x^3 \cos x\,dx$. Which of the following could be $f(x)$?

(A) $\cos x$ (B) $\sin x$ (C) $4x^3$ (D) $-x^4$ (E) x^4

25. $\int_1^{\infty} xe^{-x^2}\,dx$ is

(A) $-\frac{1}{e}$ (B) $\frac{1}{2e}$ (C) $\frac{1}{e}$ (D) $\frac{2}{e}$ (E) divergent

GO ON TO THE NEXT PAGE.

26. What is the slope of the line tangent to the polar curve $r = 1 + 2\sin\theta$ at $\theta = 0$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

GO ON TO THE NEXT PAGE.

27. For what values of p will both series $\sum_{n=1}^{\infty} \frac{1}{n^{2p}}$ and $\sum_{n=1}^{\infty} \left(\frac{p}{2}\right)^n$ converge?

(A) $-2 < p < 2$ only

(B) $-\frac{1}{2} < p < \frac{1}{2}$ only

(C) $\frac{1}{2} < p < 2$ only

(D) $p < \frac{1}{2}$ and $p > 2$

(E) There are no such values of p.

28. Let g be a continuously differentiable function with $g(1) = 6$ and $g'(1) = 3$. What is $\lim_{x \to 1} \dfrac{\int_1^x g(t)\,dt}{g(x) - 6}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) The limit does not exist.

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.